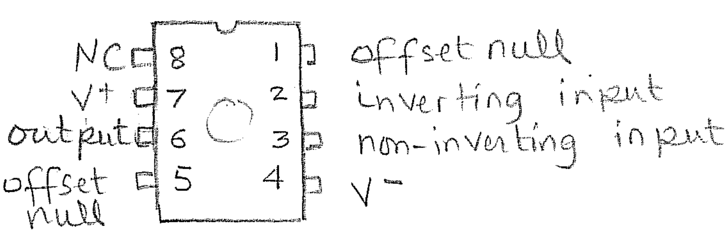
5. The Operational Amplifier.

In this chapter we will focus on the terminal behavior of the operational amplifier or "op amp". Its internal analysis requires an understanding of electronic devices such as diodes and transitors.

Yet by understanding its terminal behavior we may use op amps and resistors to build circuits that perform very useful functions such as scaling, adding, sign changing, and subtracting. In Chapter 6 we will learn how to build integrating and differentiating circuits using op amps with inductors and capacitors.

5.1 Op Amp Terminals

We will start by describing the terminals of a commercially available device the μA741 by Fairchild Semiconductors. Its top view is shown below:



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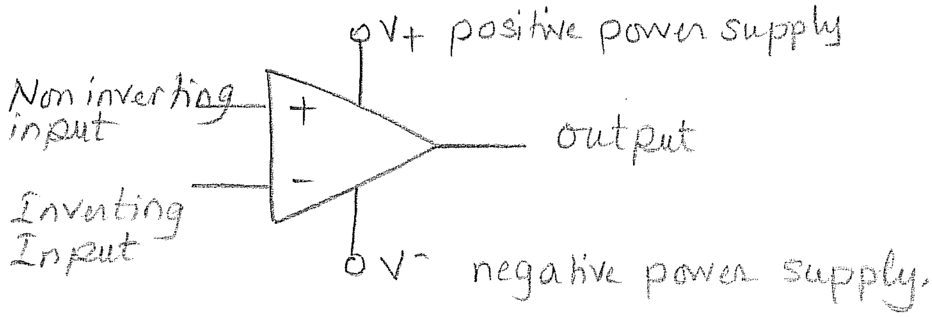
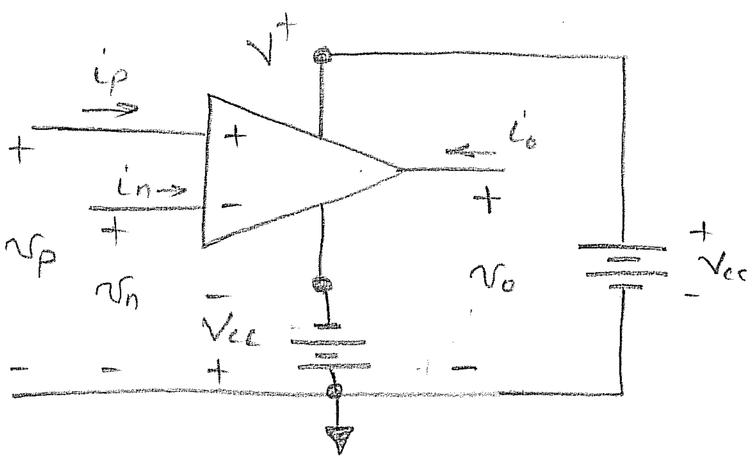
The symbol of the op amp and its terminals of primary interest are:



The other terminals are of little concern to us now.

5.2 Terminal Voltages and Currents

The terminal voltages and currents are illustrated in the figure below:



The output voltage is given by:

$$v_o = \begin{cases} -V_{cc} & A(v_p - v_n) < -V_{cc} \\ A(v_p - v_n) & -V_{cc} \leq A(v_p - v_n) \leq V_{cc} \\ +V_{cc} & A(v_p - v_n) > V_{cc} \end{cases}$$

A is usually very big of the order of 10^5. The

supply voltage is usually less than 20V. So when $10^5(v_p - v_n) < -20$ or $v_p - v_n < \frac{-20}{10^5}$
$v_p - v_n < 0.2\,mV$ then the output $v_o = -20V$.
Similarly when $v_p - v_n > 0.2\,mV$ then $v_o = 20V$.

Usually node voltages are of the order of few volts so $v_p - v_n > 0.2\,mV$ essentially means that when $v_p > v_n$ the opamp saturates to $V_{cc} = 20V$.

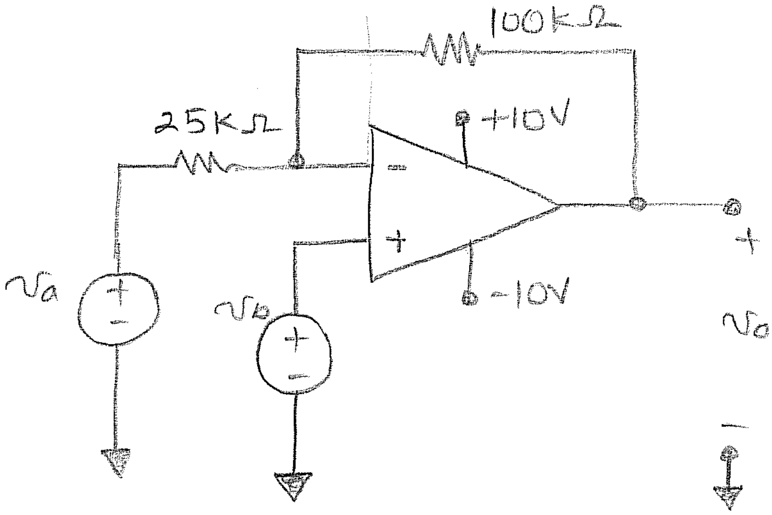
Now when the op amp operates in its linear region, e.g. 10V, then $v_p - v_n = \frac{10}{10^5} = 0.1\,mV$. So for all practical purposes $v_p \simeq v_n$! When $A \to \infty$ for an ideal op amp $v_p = v_n$. They are in a virtual short circuit. The input impedance is very high and hence the currents i_p and i_n are very small. In an ideal op amp $i_p = 0$ and $i_n = 0$!

Note that the positive and negative supply voltages are not always equal and opposites.

When the output of the op amp is connected to the inverting input, the op amp is said to have negative feedback. In this case it is likely to operate in the linear region.

Example 5.1: Analyzing an Op Amp Circuit

The op amp in the circuit shown is ideal.



a) Calculate v_o if $v_a = 1V$ and $v_b = 0V$.

Because of the negative feedback, the op amp is <u>assumed</u> to be in the linear region. thus $v_p = v_n = 0V$. Since $v_a = 1V$, then i_{25} is:

$$i_{25} = \frac{v_a - v_p}{25} = \frac{1}{25} \text{ mA}$$

and
$$i_{100} = \frac{v_o - v_p}{100} = \frac{v_o}{100} \text{ mA}$$

The current constraint $i_n = 0$ implies that:

$$i_{25} + i_{100} = \frac{1}{25} + \frac{v_o}{100} = 0 \Rightarrow v_o = -4V$$

which is in the range $\pm 10V$. So our assumption of linear operation was valid.

b) Calculate v_o if $v_a = 1V$ and $v_b = 2V$.

$$v_p = v_n = v_b = 2V$$

$$i_{25} = \frac{v_a - v_n}{25} = \frac{1-2}{25} = -\frac{1}{25} \text{ mA}$$

$$i_{100} = \frac{v_o - v_p}{100} = \frac{v_o - 2}{100}$$

$$i_{25} + i_{100} = -\frac{1}{25} + \frac{v_o - 2}{100} = 0 \Rightarrow v_o = 6V.$$

v_o lies again in the region $\pm 10V$!

c) For what range of v_b we can operate in the linear region if $v_a = 1.5V$.

From the current constraint $i_{25} = -i_{100}$ with $v_a = 1.5V$ we have:

$$\frac{1.5 - v_b}{25} = -\frac{v_o - v_b}{100}$$

solving for v_b we get:

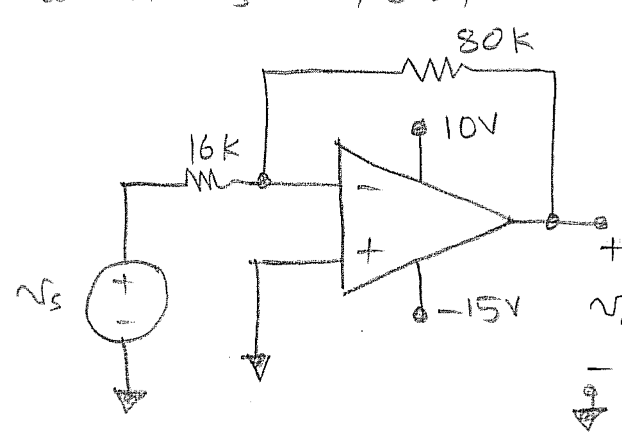
$$v_b = \frac{1}{5}(6 + v_o)$$

When v_o is in the range $-10 \leq v_o \leq 10V \Rightarrow$

$-0.8 \leq v_b \leq 3.2V$.

Assessment Problem 5.1

The op amp in the circuit shown is ideal, specify the range of v_s to avoid saturation. Determine v_o when $v_s = 2, 3.5, -1.6$ and $-2.4V$.



$$i_{16} = -i_{80} \Rightarrow \frac{v_s - v_n}{16} = -\frac{v_o - v_n}{80}$$

Note that $v_n = 0$, so:

$$v_s = -\frac{16}{80} v_o$$

For $v_o = 10V \Rightarrow v_s = -\frac{16}{80} \times 10 = -2V$

For $v_o = -15V \Rightarrow v_s = -\frac{16}{18} \times (-15) = 3V$

So $-2V \leq v_s \leq 3V$

For $v_s = 2 \Rightarrow v_o = -\frac{80}{16} \times 2 = -10V$

$v_s = 3.5 \Rightarrow v_o = -\frac{80}{16} \times 3.5 = -17.5V$

which is smaller than $-15V$. The op amp is saturated and $v_o = -15V$!

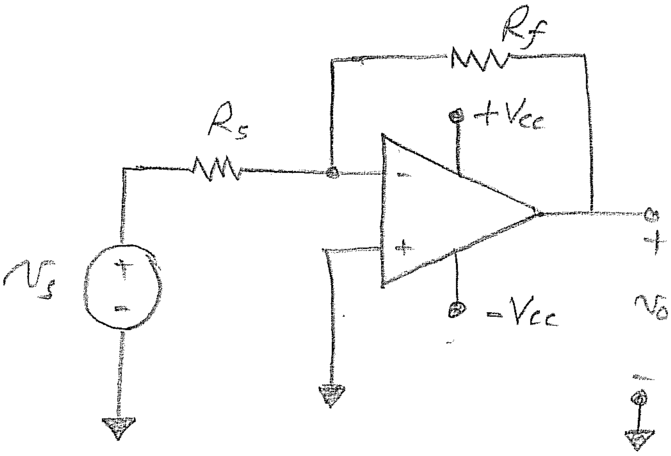
For $v_s = -1.6V \Rightarrow v_o = -\frac{80}{16} \times (-1.6) = 8V$

$v_s = -2.4V \Rightarrow v_o = -\frac{80}{16} \times (-2.4) = 12V$

which is higher than $V_{cc} = 10V$. The op amp is saturated at $+10V$ and $v_o = 10V$!

5.3 The Inverting Amplifier Circuit.

The inverting amplifier is shown below:



The circuit will be analyzed assuming an ideal op amp (i.e. $v_p = v_n$ and $i_p = i_n = 0$). The node voltage at the inverting terminal is given by $v_n = v_p = 0$! The application of KCL at the inverting terminal yields:

$$\frac{v_s - v_n}{R_s} + \frac{v_o - v_n}{R_f} = 0 \Rightarrow v_o = -\frac{R_f}{R_s} v_s$$

The output is an inverted, scaled replica of the input.

Example 5.1:

a) Design an inverting amplifier with a gain of 12. Use ±15 power supplies.

b) For what range of v_s does the op amp operate in its linear region.

a) Choose $R_s = 1\,k\Omega$ and $R_f = 12\,k\Omega$ then:

$$v_o = -\frac{R_f}{R_s} v_s = -12 v_s$$

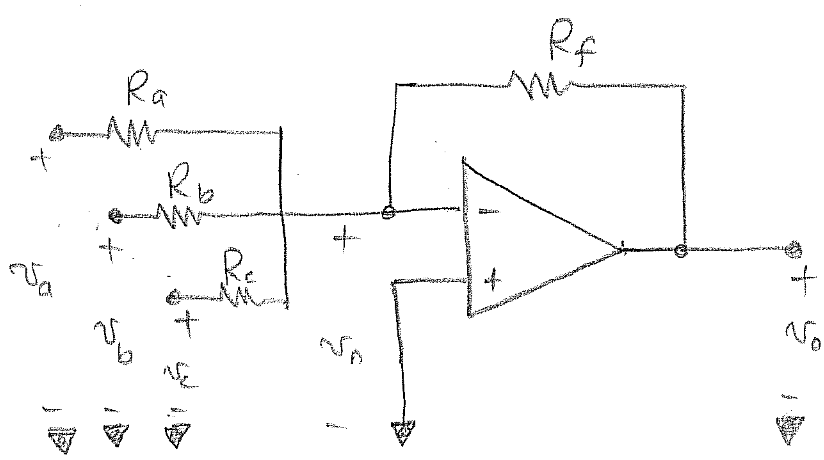
b) When $v_o = 15\,V$ then $v_s = -\frac{15}{12} = -1.25\,V$.

and for $v_o = -15$ then $v_s = -\frac{-15}{12} = 1.25\,V$.

The opamp operates in its linear region when $-1.25 < v_s < 1.25\,V$.

5.4 The Summing-Amplifier Circuit:

The output of the summing-amplifier is a scaled, inverted sum of the voltages applied at the inputs. The following diagram shows a summing amplifier with three inputs:



Write KCL at the inverting node, noting that $v_n = 0$:

$$\frac{v_a}{R_a} + \frac{v_b}{R_b} + \frac{v_c}{R_c} + \frac{v_o}{R_f} = 0 \Rightarrow$$

$$v_o = -\left(\frac{R_f}{R_a} v_a + \frac{R_f}{R_b} v_b + \frac{R_f}{R_c} v_c \right)$$

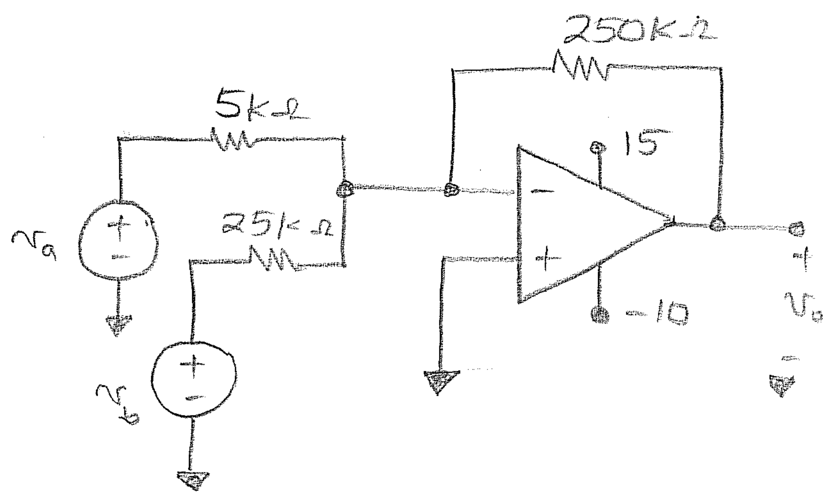
If $R_a = R_b = R_c = R_f$ then:

$$v_o = -(v_a + v_b + v_c).$$

Assessment Problem 5.3:

a) Find v_o in the circuit shown if $v_a = 0.1\,V$, and $v_b = 0.25\,V$

b) If $v_b = 0.25\,V$, how large can v_a be before the op amp saturates.



a) $$v_o = -\left(\frac{250}{5} \times 0.1 + \frac{250}{25} \times 0.25\right) \Rightarrow$$

$$v_o = -(5 + 2.5) = -7.5\,V$$

b) Because of the sign inversion as v_b increases the saturation will occur at $-10V$:

$$v_o = -10 = -\left(\frac{250}{5} \times v_a + \frac{250}{25} \times 0.25\right) \Rightarrow$$

$$v_b = 0.15\,V.$$

Problem 5.11

For the summing amplifier shown above, if $R_a = 4k\Omega$, $R_b = 5k\Omega$, $R_c = 20k\Omega$, $v_a = 200mV$, $v_b = 150mV$, and $v_c = 400mV$, then specify the range of R_f for which the op amp operates in the linear region given that $V_{cc} = \pm 6V$.

$$v_o = -R_f\left(\frac{0.2}{4} + \frac{0.15}{5} + \frac{0.4}{20}\right) \leq V_{cc}$$

$$R_f \leq -\frac{V_{cc}}{0.1} \quad \text{for } V_{cc} = -6V$$

$$R_f \leq 60k\Omega$$

The other condition should be:

$$v_o = -R_f \times 0.1 \geq -V_{cc}$$

for $V_{cc} = 6V$ we have: $R_f \times 0.1 \leq V_{cc} \Rightarrow$

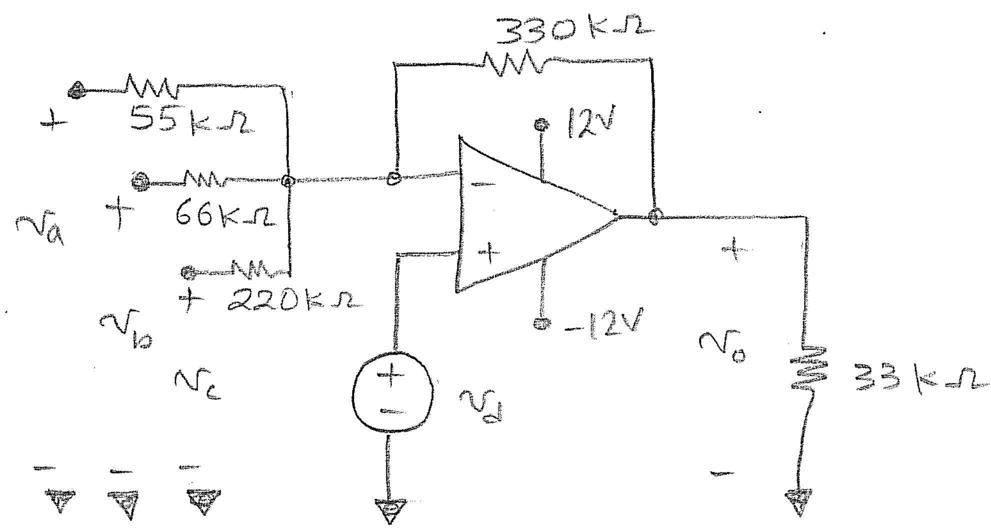
$$R_f \leq \frac{6}{0.1} = 60k\Omega$$

R_f has to be positive, i.e. non-zero, otherwise the output voltage $v_o = 0V$. A reasonable lower range is about $10k\Omega$:

$$(10k\Omega) \leq R_f \leq 60k\Omega$$

Problem 5.14

a) For the op amp circuit shown below find v_o if $v_a = 16V$, $v_b = 12V$, $v_c = -6V$ and $v_d = 10V$.



Solution:

KCL at the inverting terminal:

$$\frac{v_a - v_n}{55} + \frac{v_b - v_n}{66} + \frac{v_c - v_n}{220} + \frac{v_o - v_n}{330} = 0$$

Note that $v_n = v_p = 10V$ ($= v_d$).

$$(v_o - 10) = -\left(\frac{6}{55} + \frac{2}{66} + \frac{-6-10}{220}\right) \times 330$$

$$v_o = 10 - 22 = -12V.$$

The op amp is at the edge of saturation!

b) If v_a, v_c and v_d retain their values, specify the range of v_b such that the op amp is in the linear region.

Solution:

The first limit of v_b is 12V as can be deduced from part a). The other limit can be calculated when $v_o = 12V$.

$$12 - 10 = -\left(\frac{6}{55} + \frac{v_b - 10}{66} + \frac{-16}{220}\right) \times 330$$

So

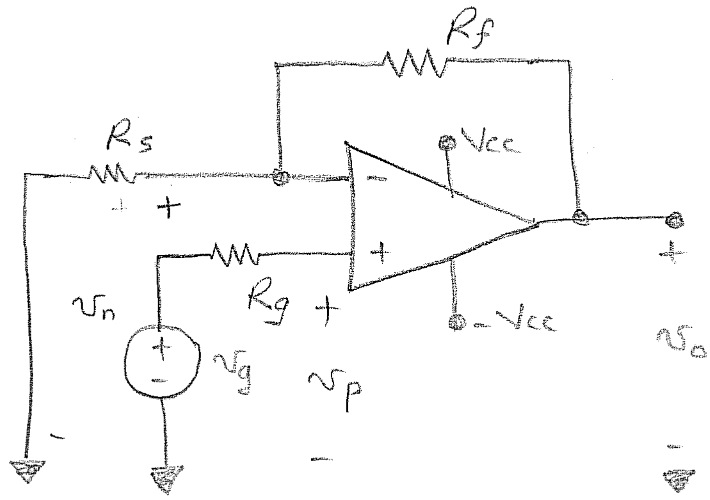
$$\frac{v_b - 10}{66} = -\frac{(12-10)}{330} - \left(\frac{6}{55} - \frac{16}{220}\right) \Rightarrow$$

$$v_b = 10 - 2 = 8V$$

$$8V \leq v_b \leq 12V$$

5.5 The Non-Inverting Amplifier Circuit.

A non-inverting amplifier circuit is shown below:



$$v_n = v_g = v_o \frac{R_s}{R_s + R_f} \Rightarrow v_o = \frac{R_f + R_s}{R_s} v_g.$$

Operation in the linear region requires that:

$$\frac{R_s + R_f}{R_s} < \left| \frac{V_{cc}}{v_g} \right|$$

Example 5.3:

a) Design a non inverting amplifier with a gain of 6.
b) If $-1.5 \le v_g \le 1.5V$, then what are the smallest power supply voltages for the op amp to remain in its linear region?

Solution:

a) $\frac{R_s + R_f}{R_s} = 6 \Rightarrow R_f = 5R_s$

If $R_f = 10k\Omega$ then $R_s = 2k\Omega$, but there is no standard resistor of $2k\Omega$. So we add two resistors of $1k\Omega$ in series at the inverting terminal.

b) $v_o = 6 v_g = 6 \times 1.5 = 9V$

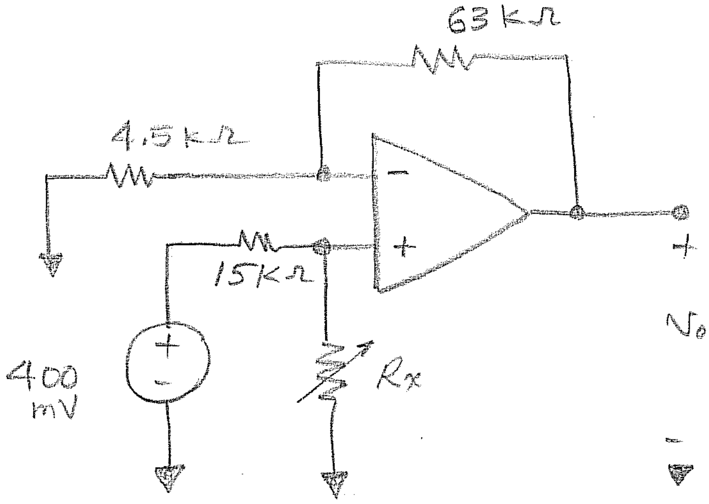
$v_o = 6 v_g = 6 \times (-1.5) = -9V$.

$V_{cc} = \pm 9V$ is the smallest voltage for which the op amp of this non-inverting circuit operates in its linear region.

Assessment Problem 5.4:

a) Find the output voltage when the variable resistor is set at 60 kΩ.

b) How large can R_x be before the op amp saturates



Solution:

a) $R_x = 60\Omega$ so $v_p = 400 \times \frac{60}{60+15} = 320\,mV$

$$v_n = v_p = v_o \frac{4.5}{4.5+63} \Rightarrow v_o = 320 \times 15 = 4800\,mV$$
$$= 4.8\,V$$

b) $$v_p = 400 \times \frac{R_x}{R_x+15} = v_o \frac{4.5}{67.5}$$

For saturation $V_o = 5V$

$$0.4 R_x = 5 \times \frac{1}{15} \times (R_x + 15) \Rightarrow$$

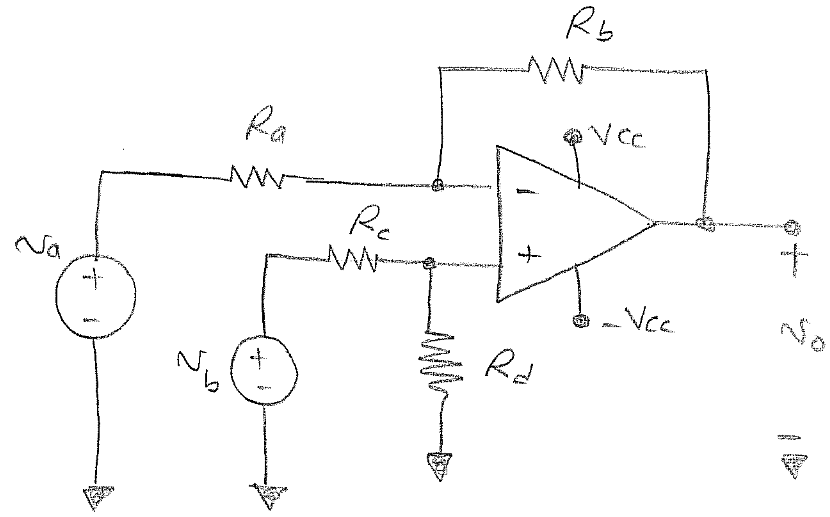
$$0.4 R_x = \frac{R_x}{3} + 5 \Rightarrow \left(0.4 - \frac{1}{3}\right) R_x = 5$$

$$R_x = \frac{5 \times 3}{0.2} = 75\,k\Omega$$

5.6 The Difference Amplifier Circuit

The output voltage of a difference amplifier is proportional to the difference between the two input voltages.

ideal op amp
$i_p = i_n = 0$
$v_p = v_n$.



By KCL at the inverting input node:

$$\frac{v_a - v_n}{R_a} + \frac{v_o - v_n}{R_b} = 0$$

But $v_n = v_p = v_b \dfrac{R_d}{R_d + R_c}$

By combining the above two equations:

$$v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} v_b - \frac{R_b}{R_a} v_a \qquad (5.22)$$

if $\dfrac{R_a}{R_b} = \dfrac{R_c}{R_d}$ then v_o reduces to:

$$v_o = \frac{R_b}{R_a}(v_b - v_a)$$

Example 5.4: Designing a difference amplifier

a) Design a difference amplifier with a gain of 8, using an ideal op amp and $\pm 8V$ power supplies.

b) For $v_a = 1V$, for what range of v_b will the op amp remain in its linear region?

Solution:

We want $\frac{R_b}{R_a} = 8 \Rightarrow R_b = 8R_a$. With $R_a = 1.5\,k\Omega$

then $R_b = 1.5 \times 8 = 12\,k\Omega$.

The ratio $\frac{R_c}{R_d} = \frac{R_a}{R_b}$ so a simple choice is $R_c = R_a = 1.5\,k\Omega$

and $R_d = R_b = 12\,k\Omega$.

b) At the edge of saturation at $V_{cc} = 8V$ we have:

$$v_o = 8 = 8(v_b - 1) \Rightarrow v_b = 2V$$

For saturation at $-8V$ we have:

$$v_o = -8 = 8(v_b - 1) \Rightarrow v_b = 0$$

So for $v_a = 1$ the op amp will be in the linear region when $0 \leq v_b \leq 2V$

* The Difference Amplifier: Measuring Performance

The performance is measured using a ratio known as the common mode rejection ratio (CMRR). We will start by redefining its inputs in terms of two other voltages:

i) The differential mode input: $v_{dm} = v_b - v_a$

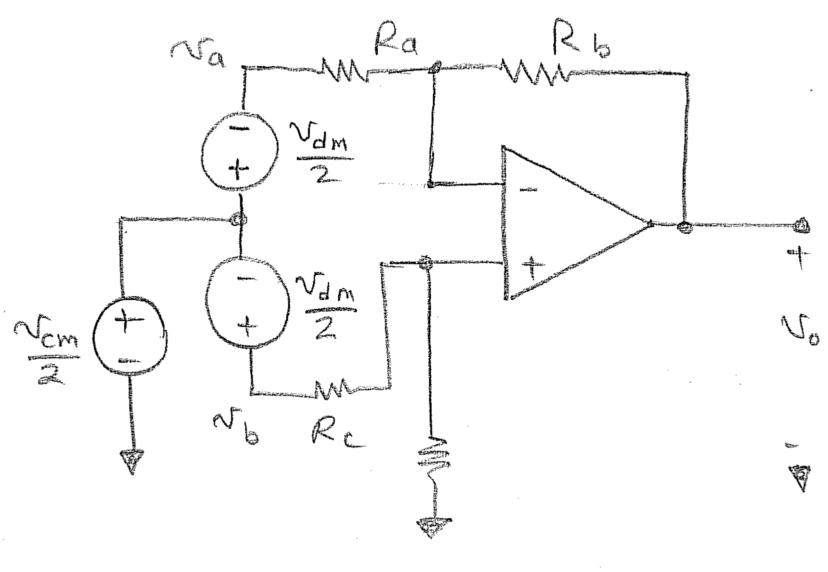
ii) The common mode input: $v_{cm} = (v_b + v_a)/2$

From the above equations we obtain:

$$v_a = v_{cm} - \frac{1}{2} v_{dm}$$

$$v_b = v_{cm} + \frac{1}{2} v_{dm}$$

These voltages (v_{dm} and v_{cm}) are illustrated in the figure below:



Replace in equation 5.22 the values of v_a and v_b given in terms of v_{cm} and v_{dm} to obtain:

$$v_o = \left(\frac{R_a R_d - R_b R_c}{R_a (R_c + R_d)} \right) v_{cm} + \left(\frac{R_d (R_a + R_b) + R_b (R_c + R_d)}{2 R_a (R_c + R_d)} \right) v_{dm}$$

$$= A_{cm} v_{cm} + A_{dm} v_{dm} \qquad (5.29)$$

When there is an exact match in the ratio $\frac{R_a}{R_b} = \frac{R_c}{R_d} \Rightarrow$ $R_a R_d - R_b R_c = 0$, then $A_{cm} = 0$. So an ideal difference amplifier has an $A_{cm} = 0$. and amplifies only the difference signal.

For example the electrocardiogram electrodes measure the heart-rate voltage which comprise the differential portion. The common-mode voltage is the noise that the electrodes pick up, which may be larger than the differential signal. In an ideal case there is no problem since $A_{cm} \to 0$, but when there is a mismatch in the ratio $\frac{R_a}{R_b} = \frac{R_c}{R_d}$ then the output signal would be mainly consist of noise.

The mismatch in the ratio of resistances can be expressed as:

$$\frac{R_a}{R_b} = (1-\varepsilon) \frac{R_c}{R_d}$$ which may be interpreted as

$$R_a = R_c \quad \text{and} \quad R_d = (1-\varepsilon) R_b \qquad (5.33)$$

Replace R_c and R_d as given by 5.33 into the expressions of A_{dm} and A_{cm} given by 5.29 and simplify to obtain:

$$A_{cm} \simeq \frac{-\varepsilon R_b}{R_a + R_b} \quad \text{and} \quad A_{dm} \cong \frac{R_b}{R_a}\left(1 - \frac{(\varepsilon/2)R_a}{R_a + R_b}\right)$$

From the above expressions we note that when there is no mismatch ($\varepsilon = 0$), then $A_{cm} = 0$ and $A_{dm} = \frac{R_b}{R_a}$!

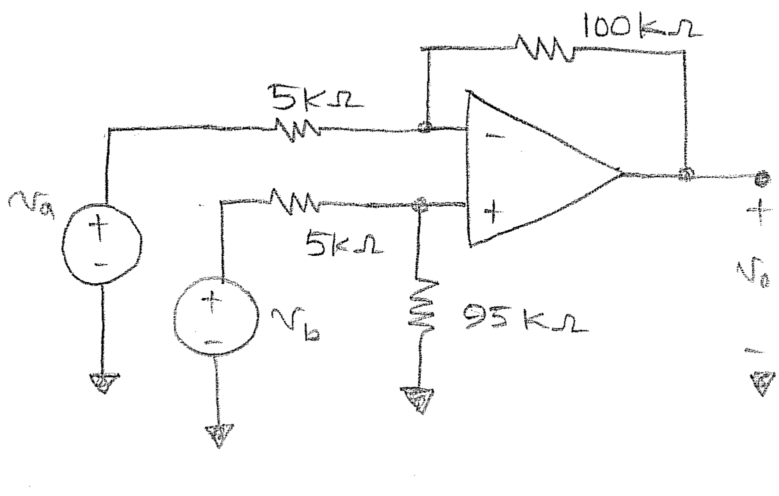
The CMRR is defined as:

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| \simeq \left|\frac{1 + R_b/R_a}{-\varepsilon}\right|$$

When the match is exact (i.e. $\varepsilon = 0$) the CMRR $\longrightarrow \infty$ i.e. the difference amplifier is amplifying the difference signal only and completely removing the noise signal.

Problem 5.34:

For the difference amplifier shown compute the a) differential-mode gain, b) the common-mode gain, and c) the CMRR.



$95 = (1-\varepsilon)\,100 \Rightarrow$

$\varepsilon = 0.05$

a) The differential mode gain is:

$$A_{dm} = \frac{100}{5}\left(1 - \frac{(0.05/2)\,5}{100+5}\right) = 19.9762$$

b) The common mode gain is:

$$A_{cm} = \frac{-0.05\;100}{100+5} = 0.04762$$

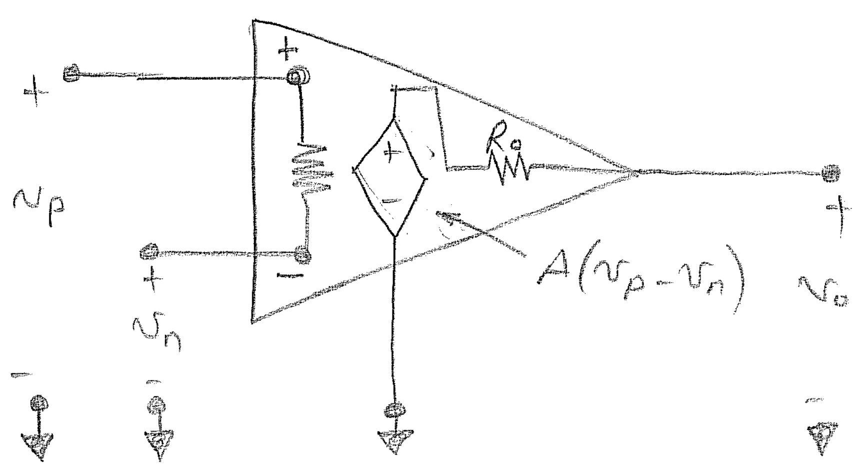
c) The common mode rejection ratio is:

$$CMRR = \frac{19.9762}{0.04762} = 419.3$$

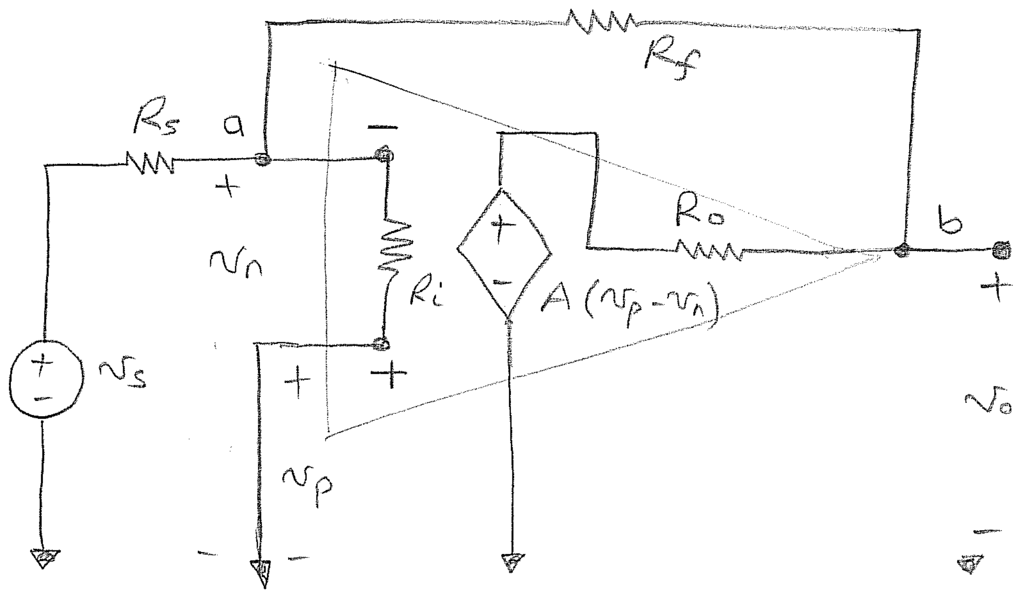
So it is not ideal!

5.7 A More Realistic Model of the Op Amp

An equivalent circuit model of the op amp is shown below. A is not infinite and Ri also is not infinite.



Let us now use the above model to analyze the inverting amplifier circuit shown below:



The circuit can be analyzed using the node-voltage method at the inverting terminal (node a) and the output terminal (node b).

node a: $$\frac{v_n - v_s}{R_s} + \frac{v_n - v_o}{R_f} + \frac{v_n}{R_i} = 0$$

node b: $$\frac{v_o - v_n}{R_f} + \frac{v_o - A(-v_n)}{R_o} = 0$$

When we rearrange the above equations we obtain:

$$\left(\frac{1}{R_s} + \frac{1}{R_f} + \frac{1}{R_i}\right) v_n - \frac{1}{R_f} v_o = \frac{1}{R_s} v_s$$

$$\left(\frac{A}{R_o} - \frac{1}{R_f}\right) v_n + \left(\frac{1}{R_f} + \frac{1}{R_o}\right) v_o = 0$$

When we solve for v_o we obtain:

$$v_o = \frac{-A + (R_o/R_f)}{\frac{R_s}{R_f}\left(1 + A + \frac{R_o}{R_i}\right) + \left(\frac{R_s}{R_i} + 1\right) + \frac{R_o}{R_f}} v_s$$

As $A \to \infty$, $R_i \to \infty$ and $R_o \to 0$ the above equation reduces to:

$$v_o = -\frac{R_f}{R_s} v_s.$$